

# A NECESSARY AND SUFFICIENT CONDITION FOR AN INFINITE BINARY STRING TO BE RECURSIVE

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Loveland and Meyer<sup>1</sup> have provided a necessary and sufficient condition for an infinite binary string to be recursive, in terms of the relative information or definitional complexity of its initial segments of length  $n$ , given  $n$ . In their notation,  $x$  is an infinite binary string for which there exists a constant  $c > 0$  such that  $K(x^n/n) \leq c$  for all  $n$ , iff  $x$  is recursive. Based on this result and other considerations we provide a necessary and sufficient condition using the absolute complexity of the initial segments, instead of the conditional complexity. An infinite binary string  $x$  is recursive iff there exists a constant  $c$  such that for all  $n$  the complexity  $K(x^n)$  of its initial segment of length  $n$  is bounded

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<sup>1</sup>D. W. Loveland, *A variant of the Kolmogorov concept of complexity*, Report 69-4, Math. Dept., Carnegie-Mellon Univ.

by  $c + \log_2 n$ .

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