**Name:** _____

1. (1 pt.)

   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this exam.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for notation, algorithmic options, etc.
   - The *last question* is for extra credit (marked $\star$).

   Write your name in the space provided above.

2. (14 pts.) Use *merge-based insertions* to insert the keys $10, 8, 6, 4, 2, 9, 7, 5, 3, 1$ into an initially empty *skew* heap. Then perform three merge-based *deleteMin* operations. Depict the state of the tree *after each operation*.

[additional space for answering the earlier question]

3. (15 pts.) Repeat all parts of Question 2 using a *pairing heap* instead of a skew heap.

[additional space for answering the earlier question]

4. (10 pts.) Trace the result of the following two *decreaseKey* operations applied to the final tree of Question 3: (1) decreasing the key 8 to 5; (2) decreasing the key 9 to 1. Recall that the structure is a pairing heap.

5. (5 pts.) Consider an undirected graph $G = (V, E)$ whose vertices are $V = \{1, 2, 3, 4, 5\}$ and whose edges are $E = \{(u, v) \mid u, v \in V,\ u \neq v\}$ (an edge between every pair of distinct vertices). The cost, or weight, of an edge in this graph is one more than the product of the numbers at its end points, modulo 5:

   $w((u, v)) = 1 + (uv \mod 5)$

   Depict $G$, labeling vertices and edges in the conventional manner

6. (5 pts.) Trace (as done in class) the operation of Dijkstra's shortest-path algorithm on the graph of Question 5, with starting vertex 3. You do not need to depict any data structures other than the graph itself.

[additional space for answering the earlier question]

7. (10 pts.) Trace the execution of *Prim's* minimum spanning tree algorithm on the graph of Question 5. You do not need to depict any data structures other than the partially formed spanning tree at each step.

8. (10 $\star$ pts.) Depict the state of the union-find data structure for each step of the trace of Question 7.

[additional space for answering the earlier question]