Name: _____

1. (1 pt.)
   - **Read all material carefully.**
   - Budget your time: 30 minutes, 30 pts ⇒ 1 min./pt. avg.
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for notation, algorithmic options, etc.
   - There is one extra-credit question at the end. It is marked with a ⋆ and is harder (and graded more strictly) than the rest.

   Write your name in the space provided above.

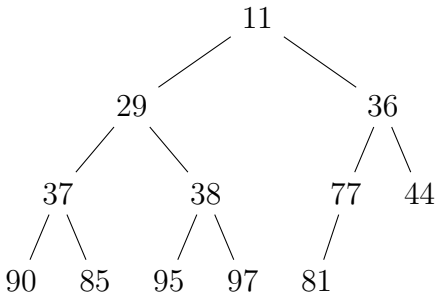
2. (9 pts.)

   (a) How many distinct (nonisomorphic) *complete binary search trees* are there containing the 11 keys: $1, 2, \ldots, 11$? (Provide an exact numerical answer.) Depict them all (or any three, if there are more than three). *Explain clearly* why the number you claim is correct.

   (b) Repeat Question 2a for *perfect binary search trees*

[additional space for answering the earlier question]

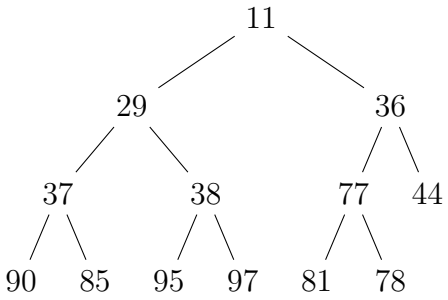
3. (10 pts.) Indicate how the key 22 is inserted into the following binary min-heap. Show all steps, as in the textbook.[1]



[1]Mark Allen Weiss, *Data Structures and Problem Solving Using Java*, 4th edition (Addison-Wesley, 2010), Figures 21.7 and 21.8.

[additional space for answering the earlier question]

4. (10 pts.) Indicate the heap that results from a *deleteMin* operation on the following binary min-heap. Show all steps, as in the textbook.[2]



[2] *Idem*, Figures 21.10–21.12.

[additional space for answering the earlier question]

5. (5 $\star$ pts.) What is the number of distinct (nonisomorphic) binary min-heaps that hold the five keys: $1, 2, 3, 4, 5$? Depict each heap in the usual graphical manner and *explain clearly why there are no others.* (If the number of heaps too large, you may abbreviate the depiction of the heaps with suitable comments to ensure the result is clear.)