Name: ______________________________

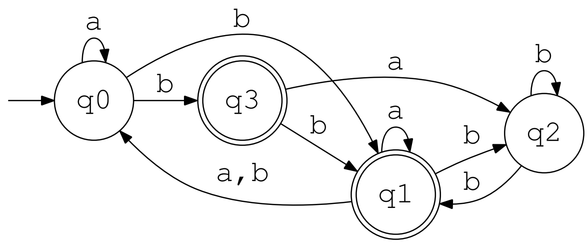
1. (1 pt.)
   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use textbook and classroom conventions for notation, algorithmic options, etc.
   - Ask for clarifications on the above if needed.
   - The question marked with a $\star$ is $\circ$ required for COS 550, but $\circ$ optional (extra credit, graded more strictly than non-$\star$) for COS 451.
   - *COS 550 students* (only) get *10 extra minutes.*

   Write your name in the space provided above.

2. (14 pts.) Recall the discussion of the *square-sum problem* on the class newsgroup and, in particular, the graph defined in its formulation. Let n denote the number of vertices in that graph. Provide:
   (a) an informal but precise prose description of the graph;
   (b) a formal definition of the graph;
   (c) the formal graph instance for $n = 14$; and
   (d) a graphical representation of the graph instance for $n = 14$.

[additional space for answering the earlier question]

3. (20 pts.) Generate a regular expression that is equivalent to the following finite-state automaton. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.



[additional space for answering the earlier question]

[additional space for answering the earlier question]

4. (20 pts.) Either (1) provide an *unambiguous* context-free grammar for the language recognized by the automaton of Question 3, and prove your claims (that the grammar generates the required language and that it is unambiguous) or (2) prove that no such grammar exists.

[additional space for answering the earlier question]

5. (15 $\star$ pts.) Define the *k-interleaved language* of languages L_1 and L_2 to be the language

$$I_k(L_1, L_2) = \{x_1 y_1 x_2 y_2 \cdots x_k y_k \quad \middle| \quad \begin{aligned} & x_1 x_2 \cdots x_k \in L_1 \wedge \\ & y_1 y_2 \cdots y_k \in L_2 \wedge \\ & \forall i \in [1, k] : x_i, y_i \in \Sigma^* \} \end{aligned}$$

and define the *interleaved language* (no k) to be the language

$$I_*(L_1, L_2) = \bigcup_{k \geq 0} I_k(L_1, L_2)$$

Prove or disprove each of the following statements separately.

(a) If L_1 and L_2 are regular then $I_k(L_1, L_2)$ is regular.
(b) If L_1 and L_2 are regular then $I_*(L_1, L_2)$ is regular.
(c) If L_1 and L_2 are regular then $I_*(L_1, L_2)$ is context-free.

[additional space for answering the earlier question]