

Name: \_\_\_\_\_

1. (1 pt.)

- **Read all material carefully.**
- Budget your time: 60 minutes, 60 pts  $\Rightarrow$  1 min./pt. avg.
- You may refer to your books, papers, and notes during this test.
- Electronic books are allowed *only when used as described in class*.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use the conventions used in class and the textbook for notation, algorithmic options, etc.
- There is one extra-credit question at the end. It is marked with a ★ and is harder than the rest (and graded more strictly).

Write your name in the space provided above.

**Wait for the go signal before proceeding to next page.**

2. (14 pts.) Depict the sequence of *bottom-up splay tree* states resulting from the insertion of the following keys, in the order presented, into an initially empty tree.

77, 89, 47, 22, 53

You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation (zig, zig-zig, zig-zag). [Hint: For each insertion, perform a standard binary-search-tree insertion, followed by a sequence of rotations to bring the new item to the root.]

[additional space for answering the earlier question]

3. (15 pts.) Repeat Question 2 for a *top-down splay tree*. In addition to the directions of that question, be sure to *clearly mark the left, middle, and right trees* using the three-column format we used in class (and from the textbook).

77, 89, 47, 22, 53

[Hint: The hint of Q. 2 does not apply here. Instead, the rotations must be performed along the way, top-down, as we proceed to the location of the item being inserted.]

[additional space for answering the earlier question]

4. (15 pts.) Repeat Question 2 for a *top-down red-black tree*. Follow the graphical conventions used in class: round nodes for red and boxed nodes for black. You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation and color change.

77, 89, 47, 22, 53

[additional space for answering the earlier question]

5. (15 pts.) Depict the action of *in-place heapsort* on the following array. Depict the *state of the array*, and the *implicit binary heap it encodes* (in the usual graphical form), after each *deleteMin* operation.

68 28 49 77 17 55



[additional space for answering the earlier question]

6.  $\boxed{\star}$  (10  $\star$  pts.) Recall the **triple-based representation of binary trees**:

We represent the empty binary tree by  $\perp$  and a nonempty binary tree with root label  $n$ , left subtree  $l$ , and right subtree  $r$  by the triple  $(n, l, r)$ .

Using this notation, define functions on binary trees that correspond to each of the following. *Explain your definitions briefly.*

(a) zig-zag

(b) skew-heap merge

[additional space for answering the earlier question]