COS 226 Fall 2018 Quiz 1 — 30 pts.; 30 minutes; 5 questions; 4 pages. — 2018-09-20 2:00 p.m.



**Name:** ______________________________________________

1. (1 pt.)
   - **Read all material carefully.**
   - Budget your **time**: 30 minutes, 30 pts. ⇒ 1 min./pt. avg.
   - You may refer to your **books**, papers, and notes during this test.
   - **No computer** or network access of any kind is allowed (or needed).
   - Write, and draw, **carefully**. Ambiguous or cryptic answers receive zero credit.
   - Use the definitions and **conventions** used in class and the textbook for notation, algorithmic options, etc.

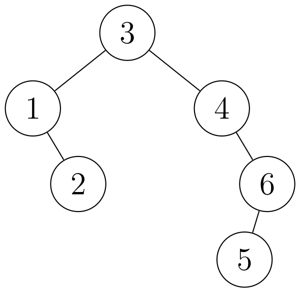
   Write your name in the space provided above.

2. (9 pts.) Determine the exact (not asymptotic) number of times the statement on line 5 (with comment *count me*) in the following code is executed.

   *Express your answer as a function* (as concise and simple as possible) of $n$ and justify it briefly. Evaluate this function for $n = 42$.

```
1   int bogo = 0;
2   for(int i = 0; i < n; i++) { // see above for 'n'
3       for(int j = 0; j < i; j++) {
4           for(int k = j; k < 5; k++) {
5               bogo = bogo + i * j; /* count me */
6           }
7       }
8   }
```

3. (10 pts.) Provide a list of keys that produce the following binary search tree when they are inserted into an initially empty tree in list order. Depict the state of the tree after each insertion.



4. (5 pts.) We represent the empty binary tree by $\perp$ and a nonempty binary tree with root $n$, left subtree $l$, and right subtree $r$ by the triple $(n, l, r)$. Consider the following function $f$ on binary trees:

$$f(t) = \begin{cases} (n, \perp, \perp) & \text{if } t = (n, \perp, \perp) \\ (n, \perp, f(l)) & \text{if } t = (n, l, \perp) \text{ and } l \neq \perp \\ (n, f(r), \perp) & \text{if } t = (n, \perp, r) \text{ and } r \neq \perp \\ (n, f(l), f(r)) & \text{if } t = (n, l, r) \text{ and } l, r \neq \perp \\ \perp & \text{otherwise} \end{cases}$$

Depict, using the usual graphical conventions, the binary tree $f(T)$ where $T$ is the tree of Question 3.

5. (5 pts.) We use the notation $f^k(t)$ (with $k > 0$) to denote $k$ nested applications of the function $f$, that is, $f(f(f(\ldots f(t))))$, where there are $k$ instances of $f$ in the expression.

   Using the definitions of $f$ and $T$ from Question 4, depict, using the usual graphical conventions, the binary trees $f^{20}(T)$ and $f^{21}(T)$. *Explain your answers.* (There is no credit for answers without proper explanations.)